Problem 12-24

The acceleration of a particle traveling along a straight line is $a = \frac{1}{4}s^{1/2}$ m/s^2, where s is in meters. If $v = 0$, $s = 1$ m when $t = 0$, determine the particle's velocity at $s = 2$ m.

Solution

The acceleration and velocity are related by

$$a = \frac{dv}{dt}.$$

Here the velocity and acceleration are functions of position, though.

$$a = \frac{dv(s)}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v$$

Substitute the given acceleration.

$$v\frac{dv}{ds} = \frac{1}{4}s^{1/2}$$

Solve for $v(s)$ by separating variables.

$$v\,dv = \frac{1}{4}s^{1/2}\,ds$$

Integrate both sides.

$$\int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} \frac{1}{4}s^{1/2}\,ds$$

Use the given initial values.

$$\int_0^v v'\,dv' = \int_1^s \frac{1}{4}s'^{1/2}\,ds'$$

Evaluate the integrals.

$$\frac{1}{2}v^2 = \frac{1}{4}\cdot\frac{2}{3}s'^{3/2}\bigg|_1^s$$

$$\frac{1}{2}v^2 = \frac{1}{6}(s^{3/2} - 1)$$

$$v^2 = \frac{1}{3}(s^{3/2} - 1)$$

$$v(s) = \pm\sqrt{\frac{1}{3}(s^{3/2} - 1)}$$

Because the acceleration is never negative, the velocity is never negative either since it starts at zero at $t = 0$.

$$v(s) = \sqrt{\frac{1}{3}(s^{3/2} - 1)}$$

Therefore, the particle's velocity at $s = 2$ m is

$$v(2) = \sqrt{\frac{1}{3}(2^{3/2} - 1)} \approx 0.781 \text{ m}.$$